CONDITIONS OF BOUNDARY-LAYER SEPARATION DURING CONDENSATION ON A CYLINDER IN A TRANSVERSE STREAM

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The conditions are analyzed under which a vaporous boundary layer separates from a cylinder in a transverse stream, with the stabilizing effect of condensation taken into account. The trend of the separation angle as a function of the heat transfer parameters is also revealed.

During condensation of vapor from a stream at the outside surface of a transversely oriented cylinder, a separation of the boundary layer causes a sharp decrease in the rate of heat transfer along the cylinder surface behind the separation point. This is attributed to two causes: first of all, behind the separation point vapor begins to flow over the film backward and thus to build up this film. Secondly, the static pressure level established behind the separation point is lower than the mainstream pressure and this causes a reduction of the effective temperature difference.

The reciprocal effect is just as significant, namely the effect of condensation on separation of the boundary layer. The condensation process, intimately related to the mass flow across the interphase boundary, is analogous to suction of the boundary layer in that it stabilizes the vapor layer and thus delays its separation from the wetted surface. All this indicates how essential it is to determine the separation angle during condensation (Fig. 1).

A theoretical basis for determining the separation angle during suction has been developed by several authors [1, 2].

The simplest criterion for separation

$$-\beta = 0.11 f_w^2 + 0.44 f_w \tag{1}$$

has been stated in [1] and its validity has been confirmed in [2].

In the case of a cylinder transversely immersed in a stream of condensing vapor, the quantities in (1) become

$$\beta = \frac{2\cos\varphi}{1+\cos\varphi}, \tag{2}$$

$$f_w = \frac{1}{2} C_Q \frac{\mathrm{Re}^{*1/2}}{\cos\frac{\varphi}{2}}. \tag{3}$$

according to Eqs. (3) and (11) of [1].

Several factors were taken into account in the derivation of formulas (2) and (3), namely: 1) a suction effect in the vaporous boundary layer due to condensation, 2) a velocity distribution on the outer edge of the vaporous boundary layer, in accordance with the law for a cylinder in a stream of an ideal fluid, and 3) a small compressibility effect, making $u_e^2/2h_{se} \ll 1$ in Eq. (3) in [1].

Solving Eq. (1) for f_w and inserting the values from (2) and (3), we obtain

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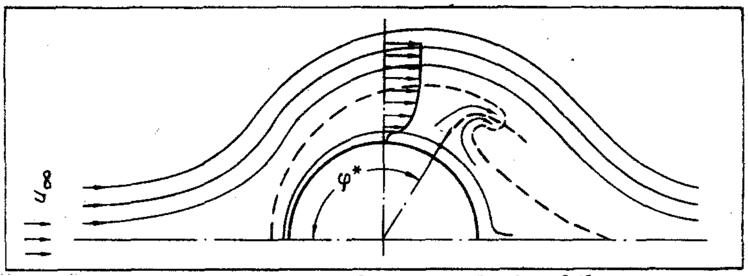

Fig. 1. Schematic diagram for an analysis of the process.

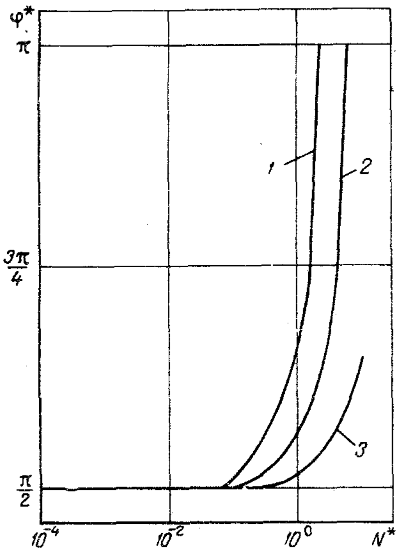

Fig. 2. Separation angle φ^* as a function of the number N^*, for 1) $Re'' = 10^2$; 2) $Re'' = 10^3$; 3) $Re = 10^4$.

$$\frac{C_Q}{2\cos\frac{\varphi}{2}}\,Re''^{1/2} = -2 + \sqrt{4 - \frac{18.2\cos\varphi}{1+\cos\varphi}}\,. \tag{4}$$

Formula (4) defines the condition of separation during condensation at the cylinder surface. The second solution (with a minus sign before the square root) has been discarded as physically meaningless. (The quantity C_Q is always positive under the conditions stipulated here.)

For a transverse flow around a cylinder under conditions of weightlessness, we introduce the heat transfer coefficient according to [3]:

$$\alpha_\varphi = \frac{\sin\varphi}{\sqrt{1-\cos\varphi}}\sqrt{\frac{\lambda^2\rho u_\infty}{\mu D}} = \cos\frac{\varphi}{2}\,\frac{\lambda}{D}\sqrt{2Re},$$

and obtain an explicit expression for the separation angle:

$$\cos\varphi = -\frac{N^2\left(\frac{\rho}{\rho''}\,\frac{\mu}{\mu''}\right) + 4\sqrt{2}N\left(\frac{\rho}{\rho''}\,\frac{\mu}{\mu''}\right)^{1/2}}{N^2\left(\frac{\rho}{\rho''}\,\frac{\mu}{\mu''}\right) + 4\sqrt{2}N\left(\frac{\rho}{\rho''}\,\frac{\mu}{\mu''}\right)^{1/2} + 36.4}\,. \tag{5}$$

The minus sign in (5) indicates that the separation angle is in this case $90° < \varphi < 180°$.

With the effect of the gravitational field (condensation of vapor flowing downward along the surface of a horizontally oriented cylinder) taken into account, it is impossible to obtain an explicit analytical expression for the separation angle. We find the separation angle from (4), instead, namely:

$$\cos\frac{\varphi}{2} = \frac{1}{4.55}\left[-\frac{Nu_\varphi N^*}{4\,Re''^{1/2}} + \sqrt{10.35125 - 3.55\left(\frac{Nu_\varphi N^*}{4\,Re''^{1/2}}\right)^2}\right]. \tag{6}$$

Since Nu_φ is a function of the angle φ, the latter can be determined from (6) by a numerical method only.

With the aid of a graph representing this relation at fixed values $Nu_\varphi = 25$ and $Re'' = 10^2$, 10^3, or 10^4 within the range $10^{-4} \le N^* \le 10$, we plot curves for the separation angle as a function of N^* (Fig. 2).

The curves indicate that, approximately up to $N^* = 10^{-1}$, separation occurs at angles $\varphi^* \approx 90°$. This is the lower limit of separation angles, which corresponds to an absence of condensation (suction). Obviously, this result is a consequence of initially assuming the laws of potential flow around a cylinder, and it differs somewhat from the well known test results ($\varphi^* \approx 82°$) [4]. Formula (6) is structured so that the requirement $\cos\varphi/2 \ge 0$ makes the magnitude of the complex group $(Nu_\varphi N^*/Re''^{1/2})^2 \le 36.4$, which then defines the range where the separation angle depends on the thermophysical parameters in this complex group.

When $(Nu_\varphi N^*/Re''^{1/2})^2 > 36.4$, Eq. (6) first yields separation angles larger than 180° and then has no real solution for $(Nu_\varphi N^*/Re''^{1/2})^2 > 46.65$. Since $(Nu_\varphi N^*/Re''^{1/2})^2 > 36.4$ represents an increasing suction effect, hence it is physically obvious that the condensation rate is then more than sufficiently high to completely prevent separation of the vaporous boundary layer. The separation angle under such conditions is, naturally, independent of the condensation rate and always equal to 180°.

NOTATION

β	is a parameter which characterizes the pressure gradient;
f_w	is a parameter which characterizes the suction rate;
φ	is an angle measured from the frontal line of the cylinder;
φ^*	is the separation angle;
u_∞	is the vapor velocity far away from the cylinder;
v	is the normal component of velocity;
D	is the cylinder diameter;
μ	is the dynamic viscosity of the liquid;
ν	is the kinematic viscosity of the liquid;
μ''	is the dynamic viscosity of the vapor;
ν''	is the kinematic viscosity of the vapor;
$C_Q = -v/u_\infty = \alpha_\varphi \Delta T / r\rho'' u_\infty$	is the dimensionless flow-rate coefficient;
$Re = u_\infty D/\nu$	is the modified Reynolds number;
$Re'' = u_\infty D/\nu''$	is the Reynolds number of the vapor stream;
ρ	is the density of the liquid;
ρ''	is the density of the vapor;
$N = \lambda\Delta T/r\mu,\ N^* = \lambda\Delta T/r\mu''$	are dimensionless numbers;
α_φ	is the local coefficient of heat transfer;
r	is the latent heat of condensation;
ΔT	is the temperature difference between cooling wall and saturated vapor;
λ	is the thermal conductivity of the liquid;
Nu_φ	is the local Nusselt number.

LITERATURE CITED

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